THURSDAY, JUNE 8, 1905.

A MANUAL OF QUATERNIONS.

A Manual of Quaternions. By Prof. Charles Jasper Joly, F.R.S. (London : Macmillan and Co., Ltd., 1905.) Pp. xxvii + 320. Price 10s. net.

PROF. C. J. JOLY'S " Manual of Quaternions " is an important addition to the literature of the subject. It at once takes rank with Tait's " Treatise " as an eminently serviceable exposition of Hamilton's great calculus.

Hamilton's own works, the " Lectures " and the " Elements," are in their way inimitable. Unfortunately, their style is not suited to the average student eager to acquire a working knowledge of the mathematical method developed in them. Tait alone of the younger contemporaries of Hamilton seemed to have been able to appreciate the " Lectures "; but he himself used to relate how, as he laboriously read through the first six, he began to despair of his own powers. There seemed to be such diffuse discussion, and withal so little apparent progress. But the seventh lecture came like a transformation scene. Every page displayed new beauties, every paragraph disclosed the marvellous power and variety of the method. From it Tait drew his inspiration, and proceeded to enlighten the world as to the meaning and purpose of the quaternion.

To the student who has grasped the essentials of the method Hamilton's second volume, the " Elements," will always prove a happy hunting ground; but experience has shown that its very completeness acts as a deterrent. In the much smaller treatise written by Tait, the important practical aspects of quaternions are more rapidly though less logically developed, and the chief value of Tait's work lies in his characteristic treatment of dynamical and physical problems. It has been long felt, however, that a good working manual of quaternions was needed, by use of which the mathematical student could come quickly into touch with all that is essential in the calculus. This is what Prof. Joly has endeavoured to supply.

For reasons clearly explained in the preface, the author has (reluctantly, he confesses) forsaken the Hamiltonian approach. Instead of developing the calculus logically from the definition of a quaternion as the ratio of two vectors, he defines independently the quantities $S\alpha\beta$ and $V\alpha\beta$, and then writes the product $\alpha\beta$ as equal to the sum of these two. The student must, of course, take on trust that there is some good reason for defining $S\alpha\beta$ as *minus* the product of the length of one vector into the length of the projection of the other upon it. This is, at root, the peculiarity of Hamilton's system which troubled O'Brien nearly sixty years ago, and has not ceased to trouble occasional critics since. There is a kind of notion hovering about in some minds that the positive sign in algebra is more natural than the negative sign, the truth being, of course, that the one necessarily implies the other. It is to be feared, however, that this apparently arbitrary assumption of the negative sign in translating $S\alpha\beta$ into ordinary trigonometrical notation (Clifford calls it a

convention) will puzzle many a student. Prof. Joly soon gives the reason for the negative sign, though not quite so definitely as might be desirable; but it is questionable if its full significance will be appreciated until considerable progress has been made in acquiring quaternionic skill. The reader is advised to exercise a strong faith, and to proceed nothing fearing. If he persevere he will soon get out of the valley of the shadow of the negative sign.

It is possible that some critic may regard this forsaking of Hamilton's logical basis as a confession of weakness. But this is not so. The weakness is in the average student, for whom a somewhat simple intellectual diet must be prepared in the hope that the mental digestion may be strengthened sufficiently to assimilate the strong Hamiltonian food which Prof. Joly serves up a little later. The truth is that very few students are able to appreciate to the full an absolutely logical argument until they have a certain amount of practical knowledge imparted to them more or less by authority.

So far as the *principle* of the method is concerned, Prof. Joly ranges himself at first on the side of those vector analysts who neglect the quaternion. But it is only for a couple of pages at the beginning of chapter ii. On p. 8 the important formula

$$(\alpha\beta = S\alpha\beta + V\alpha\beta)$$

is introduced as a definition of the quaternion, and the quaternion is never afterwards lost sight of. Its fundamental importance and analytic value are in evidence on every page. It must be admitted that by this line of approach the reader is rapidly brought into touch with the essential elements of the subject. There is, nevertheless, a certain arbitrariness which is not satisfying to the mind, nor is it clear when all is done what is really fundamental. A critically minded student might possibly be inclined to say, Why not define $S\alpha\beta$ as *plus* the product of the lengths of the vectors into the cosine of the angle between them, and then define the quaternion $\alpha\beta$ by the formula $V\alpha\beta - S\alpha\beta$? At first sight it seems to amount to the same thing, and yet, as will be found on trial, it leads to a system clothed in quaternion garments, but more like the fabulous ass in the lion's skin than the real lion.

Having thus established in chapter ii. the fundamental properties of the quaternion, Prof. Joly rapidly runs over certain important transformations of vector products and ratios (chapters iii. and iv.), and simple applications to the geometry of the straight line, plane and circle (chapters v. and vi.). Then follow, treated in separate chapters, differentiation, linear vector functions, quadric surfaces, and the geometry of curves and surfaces. Here the *power* of the calculus asserts itself strongly. Numerous examples are supplied throughout for the student to work upon and develop his analytical skill. In subsequent chapters dynamical problems of various kinds are taken up—such as astatic equilibrium, screws and wrenches, strains, central forces, constrained motion, motion of a rigid body, and the like. A valuable and well arranged chapter on the operator ∇ treats of heterogeneous strain, spherical harmonics, hydrodynamics, elasticity, electromagnetic theory, and wave propagation generally. The

G

treatment is by no means superficial, and is in many places highly condensed. It is all done in forty-two pages, a remarkable testimony to the compactness of quaternion notation and the brevity of quaternion proofs. In chapter xvii., on projective geometry, Prof. Joly gives his own interesting extension, in which a new interpretation is assigned to the quaternion, and he concludes in chapter xviii. with quaternions generalised so as to be applicable to space of any number of dimensions.

There can be no question as to the high merits of the " Manual of Quaternions." It is a worthy companion volume to the master's own great works. Like the " Elements " of Hamilton and the " Elementary Treatise " of Tait, it is characterised by the extraordinary range of mathematical subjects which come within its scope. It is not merely the substitution of one symbol for three or one for four which makes this condensation possible, for that, after all, is a question simply of notation. But the quaternion calculus rejoices in the possession of two remarkable operators, the linear vector function ϕ and the vector differentiator ∇. They operate singly and in combination according to laws which naturally evolve themselves from the fundamental laws of the calculus. They can be linked together in an endless variety of ways, and go far to give to Hamilton's quaternions a flexibility, power, and pictorial compactness not possessed by any other general method which is directly applicable to problems of mathematics pure and applied. These features are exquisitely brought out in Prof. Joly's " Manual."   C. G. K.

SOME MEDICAL WORKS.

(1) *New Methods of Treatment.* By Dr. Laumonier. Translated from the second revised and enlarged French edition, and edited by Dr. H. W. Syers. Pp. xvii + 321. (London : Constable and Co., Ltd., 1904.) Price 7s. 6d. net.

(2) *The Surgery of the Diseases of the Appendix Vermiformis and their Complications.* By W. H. Battle and E. M. Corner. Pp. xi + 208. (London : Constable and Co., Ltd., 1904.) Price 7s. 6d. net.

(3) *Clinical and Pathological Observations on Acute Abdominal Diseases.* (The Erasmus Wilson Lectures, 1904.) By E. M. Corner. Pp. 98. (London : Constable and Co., Ltd., 1904.) Price 3s. 6d. net.

(4) *A Short Treatise on Anti-Typhoid Inoculation.* By Dr. A. E. Wright. Pp. x + 76. (London : Constable and Co., Ltd., 1904.) Price 3s. 6d. net.

(5) *The Suppression of Tuberculosis.* By Prof. E. von Behring. Authorised translation by Dr. Charles Bolduan. Pp. v + 85. (New York : John Wiley and Sons; London : Chapman and Hall, Ltd., 1904.) Price 4s. 6d. net.

(1) EVERY year a multitude of substances, chiefly synthetic, is introduced, every one being extolled as a certain cure for this or that ailment. By good fortune one of them is now and then found to be of real value, and for a time at least finds a place in the "aramentaria medica," but the majority in a year or two pass into oblivion. Similarly new modes of treatment come and go, most of them being of little worth. But the medical practitioner is expected to know of all these medicinal substances and vagaries of treatment, and must be prepared to employ any one of them at the suggestion of some faddist who happens to consult him. From this point of view the first book on our list may be a useful guide, but otherwise one would be inclined to ask, *cui bono?* Some of the substances included in the volume are by no means new, *e.g.* thyroid, guaiacol, and the antitoxic sera, while others which have a greater claim to novelty, and are, moreover, of real value, such as aspirin, acetozone, urotropine and cystamine, and purgen, are omitted. In dealing with tetanus antitoxin, no mention is made of injection into the spinal cord or nerve trunks. As regards phosphorised principles, lecithin, glycerophosphates, &c., which have of late been extolled in wasting diseases and nervous affections, the administration of a couple of eggs a day would probably be of far greater benefit than any of the medicinal preparations of these substances.

(2) Messrs. Battle and Corner give a succinct account of the anatomy, pathology, symptoms, and treatment of that common and fashionable malady appendicitis which may be safely recommended as a guide for the medical practitioner. The origin and function of the vermiform appendix are discussed, that little blind tubular appendage of the bowel inflammation of which gives rise to so much trouble. The appendix has usually been regarded as a vestigial structure and useless in function, but the researches of Mr. R. Y. A. Berry, of Edinburgh, suggest that it is a specialised mass of lymphoid tissue which the authors conceive may serve as a defensive mechanism against bacterial invasion in a portion of the bowel where, for anatomical and other reasons, there is a delay in the passage of the intestinal contents onwards, and special protection is therefore required against the absorption of bacterial products.

(3) This work is based on material collected in compilation of the Erasmus Wilson lectures, 1904. The author states that the main object of his lectures was to direct attention to the identity of the pathological changes concerned in the production of all acute perforative and gangrenous processes of the alimentary tract. He suggests that two extremes of tissue death or necrosis may be recognised, viz. that due to deprivation of blood and that caused by the action of micro-organisms. Between these two there are various grades and admixtures; the former is slow in action, the latter very rapid, and it is this which plays so important a part in abdominal necrosis. The work is practically a collection of notes, but is interesting reading.

(4) Prof. Wright has done well to collect into a single volume the various papers, with amplifications, he has from time to time contributed to various journals on the subject of anti-typhoid vaccination. The method of preparation of the vaccine, theoretical and practical considerations as to its use, and statistics of its value are all considered. With regard to the last named, it must be mentioned that some controversy has taken place in the medical Press as to